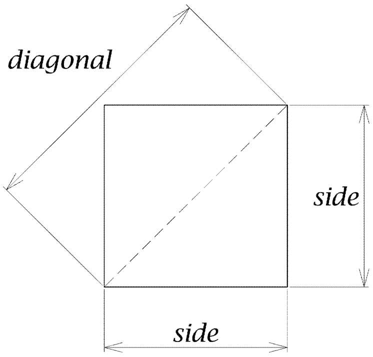
Mensuration the study of geometric figures

Mensuration is the branch of mathematics which deals with the study of various parameters of geometric figures such as areas, perimeters, volumes etc.

Square geometric figure



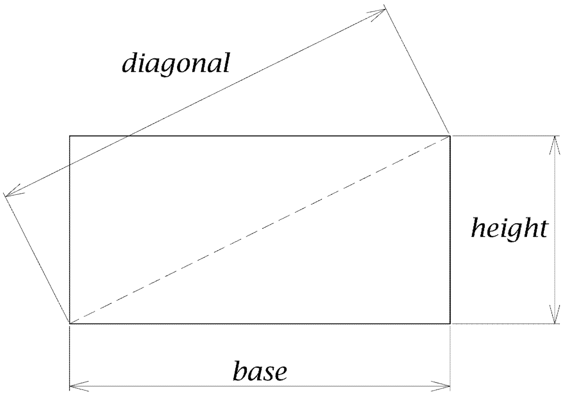
$side := 10.0\ mm$

$area := side^2 = 100.0\ mm^2$

$perimeter := 2 \cdot (side + side) = 40.0\ mm$

$diagonal := \sqrt{(2)} \cdot side = 14.1421\ mm$

Rectangle geometric figure



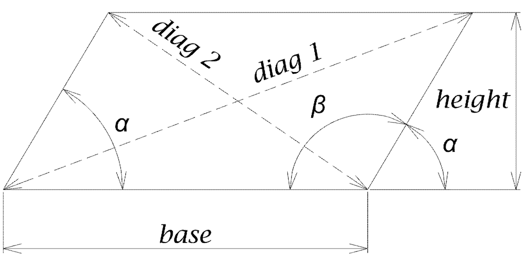
$height := 10.0\ mm$ $base := 20.0\ mm$

$area := height \cdot base = 200.0\ mm^2$

$perimeter := 2 \cdot (height + base) = 60.0\ mm$

$diagonal := \sqrt{(height^2 + base^2)} = 22.3607\ mm$

Parallelogram geometric figure



$height := 10.0\ mm$ $base := 20.0\ mm$ $\alpha := 60.0\ deg$

angle beta = 180 deg - angle alpha

$\beta := 180\ deg - \alpha = 120.0\ deg$

angles converted to radians

$\alpha = 1.0472\ rad$ $\beta = 2.0944\ rad$

side adjacent to angle alpha found using trigonometry

$adj := \frac{height}{tg(\alpha)} = 5.7735\ mm$

length of sloping side found using Pythagorean theorem

$slope := \sqrt{adj^2 + height^2} = 11.547\ mm$

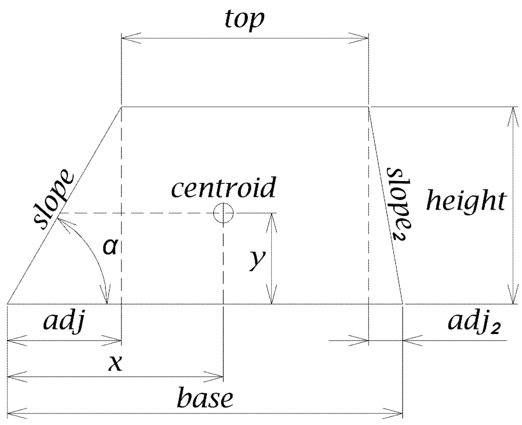
$area := height \cdot base = 200.0\ mm^2$

$perimeter := 2 \cdot (slope + base) = 63.094\ mm$

$diag\_1 := \sqrt{(slope^2 + base^2) + 2 \cdot (slope \cdot base \cdot \cos(\alpha))} = 27.6455\ mm$

$diag\_2 := \sqrt{(slope^2 + base^2) - 2 \cdot (slope \cdot base \cdot \cos(\alpha))} = 17.3895\ mm$

Trapezium geometric figure



$height := 10.0\ mm$ $base := 20.0\ mm$ $top := 12.5\ mm$ $\alpha := 60.0\ deg$

angle converted to radians

$\alpha = 1.0472\ rad$

side adjacent to angle alpha found using trigonometry

$$adj := \frac{height}{tg(\alpha)} = 5.7735\ mm$$

$$adj_2 := base - (top + adj) = 1.7265\ mm$$

length of sloping sides found using Pythagorean theorem

$$slope := \sqrt{adj^2 + height^2} = 11.547\ mm$$

$$slope_2 := \sqrt{adj_2^2 + height^2} = 10.1479\ mm$$

$$area := \frac{height \cdot (base + top)}{2} = 162.5\ mm^2$$

$$perimeter := top + base + slope + slope_2 = 54.195\ mm$$

To determine the x and y co-ordinates of the centroid

$$x := \frac{base}{2} + \frac{(2 \cdot top + base) \cdot \left(slope^2 - slope_2^2\right)}{6 \cdot \left(base^2 - top^2\right)} = 10.9339\ mm$$

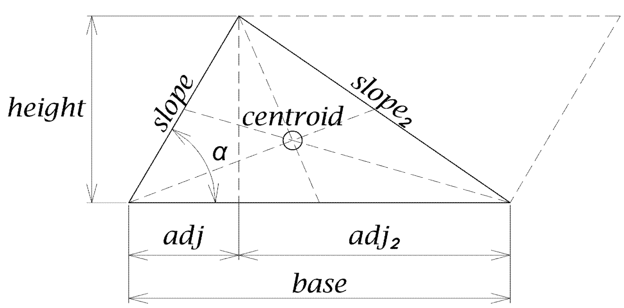
$$y := \frac{base + 2 \cdot top}{3 \cdot (top + base)} \cdot height = 4.6154\ mm$$

To determine the x and y co-ordinates from the top of the Trapezoid and right edge

$$x_1 := \frac{base}{2} - \frac{(2 \cdot top + base) \cdot \left(slope^2 - slope_2^2\right)}{6 \cdot \left(base^2 - top^2\right)} = 9.06608\ mm$$

$$y_1 := \frac{top + 2 \cdot base}{3 \cdot (top + base)} \cdot height = 5.3846\ mm$$

Triangle geometric figure



$height := 10.0\ mm$ $base := 20.0\ mm$ $\alpha := 60.0\ deg$

angle converted to radians

$\alpha = 1.0472\ rad$

side adjacent to angle alpha using trigonometry

$$adj := \frac{height}{tg(\alpha)} = 5.7735\ mm$$

$$adj_2 := base - adj = 14.2265\ mm$$

length of sloping sides using Pythagorean theorem

$$slope := \sqrt{adj^2 + height^2} = 11.547\ mm$$

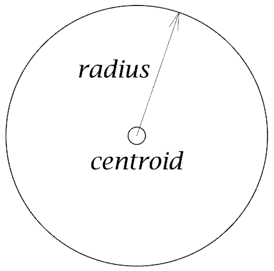
$$slope_2 := \sqrt{adj_2^2 + height^2} = 17.3895\ mm$$

$$area := \frac{height \cdot base}{2} = 100\ mm^2$$

$$perimeter := base + slope + slope_2 = 48.9365\ mm$$

The centroid of the triangle is the point where the triangle's three medians intersect.

Circle geometric figure



π is the ratio of a circles circumference to its diameter

$radius := 10.0\ \text{mm}$

$\pi = 3.141593$

$area := \pi \cdot radius^2 = 314.1593\ \text{mm}^2$

$perimeter := 2 \cdot \pi \cdot radius = 62.8319\ \text{mm}$

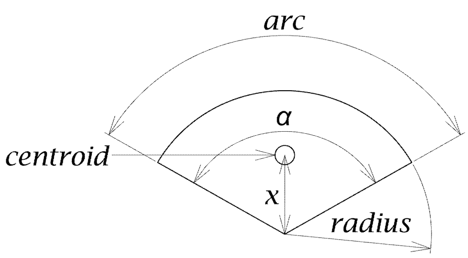
The centroid of a circular figure is a point equidistant from any point on the perimeter of the circle, (the centre of the circle)

$diameter := 2 \cdot radius = 20\ \text{mm}$

$circumference := perimeter = 62.8319\ \text{mm}$

$$ratio := \frac{circumference}{diameter} = 3.141593$$

Sector of circle geometric figure



$radius := 7.5\ \text{mm}$ $\alpha := 120.0\ \text{deg}$

$$area := \frac{radius^2 \cdot \alpha}{2} = 58.9049\ \text{mm}^2$$

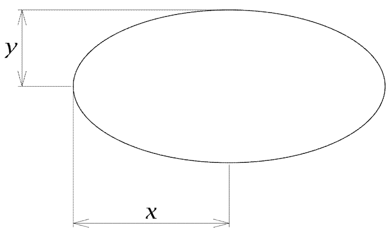
$arc := radius \cdot \alpha = 15.708\ \text{mm}$

$perimeter := 2 \cdot radius + arc = 30.708\ \text{mm}$

$$x := \frac{2 \cdot radius \cdot \sin\left(\frac{\alpha}{2}\right)}{3 \cdot \frac{\alpha}{2}} = 4.135\ \text{mm}$$

Ellipse geometric figure

$x := 10.0\ \text{mm}$ $y := 5.0\ \text{mm}$



$area := \pi \cdot x \cdot y = 157.0796\ \text{mm}^2$

The most accurate method to obtain the perimeter is Calculus based on the complete elliptic integral of the second kind. More detail is given on the Wikipedia page:

https://en.wikipedia.org/wiki/Ellipse

Hyperlink to https://en.wikipedia.org/wiki/Ellipse not available from SMath Cloud
Download sheet and use with "Hyperlink Region" by davide Carpi

$$e := \sqrt{1 - \frac{y^2}{x^2}} = 0.866$$

equation to calculate the eccentricity (e) of the Ellipse

$$perimeter := 4 \cdot x \cdot \int_0^{\frac{\pi}{2}} \sqrt{1 - e^2 \cdot (\sin(\theta))^2}\ d\theta = 48.4422\ \text{mm}$$

There are also a number of different equations for approximating the perimeter.

$perimeter := \pi \cdot (x + y) = 47.1239\ \text{mm}$

$$perimeter := 2 \cdot \pi \cdot \sqrt{\frac{y^2 + x^2}{2}} = 49.6729\ \text{mm}$$